RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2015

FIRST YEAR

PHYSICS (Honours)

Date : 21/05/2015

Time : 11 am – 3 pm

Paper : II

Full Marks : 100

[Use a separate Answer book for each group]

Group – A

[*Answer* ***any four*** *questions*]

1. a) Find the Fourier Series of the periodic function defined by,
$f(x)=0,\ -\pi \le x < 0,\ f(x)=\pi,\ 0 \le x < \pi$
Give a graph of this function, and indicate the points of discontinuity. What is the value of the function at $x=\pi$? Hence, show that $\sum_{n=1}^{\infty}\frac{1}{(2x-1)^2}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$. [4+1+1+1]

   b) Find the Fourier transform of the Gaussian function, $f(x)=e^{-\alpha x^2}$ $(\alpha>0$, constant) [3]

2. a) Use the method of separation of variables to obtain for the one-dimensional heat equation $K\frac{\partial^2 u(x,t)}{\partial x^2}=\frac{\partial u}{\partial t}$, a solution that tends to zero, as $t \to \propto$. [4]

   b) Separate Laplace's equation in spherical polar coordinates (r,θ,ϕ). Show that the solution of the ϕ-equation is given by $A\cos m\phi + B\sin m\phi$ where m is an integer. [4+2]

3. Consider the three-dimensional wave equation for $u(\vec{r},t)$, $\nabla^2 u=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$.

   a) Show that with the choice $u(\vec{r},t)=F(\vec{r})T(f)$, the equation breaks up into the two equations, $\nabla^2 F + k^2 F = 0$; $\frac{d^2T}{dt^2}+k^2e^2T=0$, when k^2 is a separation constant. [3]

   b) If F is spherically symmetric, show that u can be written as $u(r,t)=\frac{1}{r}e^{i(kr-\omega t)}$, where $\omega = kc$. [3]

   c) State Dirichlet's conditions for a function to be expanded as a Farier Series. Verify these conditions for the square wave represented by,
$$f(x)=-1,\ \frac{1}{2}\le x<0$$
$$=+1,\ 0\le x<\frac{L}{2}$$
and find its Fourier expansion. Give a graph of this function. [4]

4. a) Two particles of masses m and M interact via a central force $\vec{F}(r)=\hat{r}f(r)$. Show that the problem can be reduced to an equivalent one-body problem for a particle of reduced mass μ in the reference frame where the CM is at rest. [4]

   b) Show that the above motion can be reduced to a one-dimensional motion in an effective potential, $u_{eff}(r)$. [4]

   c) If $f(r)=-\frac{k}{r^2}(k>0)$, show that when the total energy E of the system is negative, there are two turning points of the orbit. [2]

5. a) State Keplar's laws of placatory motion, with brief explanation of each. [4]

b) Given that the orbit equation of the planet is the ellipse, $\frac{\ell}{r} = 1 + e\cos\theta$ (symbols have their usual meanings)

Show that the force law is of the form, $f(r) = -\frac{k}{r^2}(k > 0)$. Hence show that the speed v of the planet is given by, $v^2 = \frac{k}{m}\left(\frac{2}{r} - \frac{1}{a}\right)$, where a is the length of the semi-major axis. (Your may make the necessary assumptions, as required) [6]

6. a) A reference frame R rotates with uniform angular velocity $\vec{\omega}$ w.r.t an inertial frame S, having common origin O. The equation of motion of a moving particle of mass m in S, is $m\vec{a}_s \equiv m\ddot{\vec{r}} = \vec{F}$, $\vec{F}$ is the applied force.

Show that in R, the effective force $\vec{F}_{eff}$ necessary to describe the motion is given by, $\vec{F}_{eff} = m\vec{a}_R = \vec{F} - 2m\vec{\omega}\times\vec{v} - \vec{\omega}\times(\vec{\omega}\times\vec{r})$

where $\vec{v}$ is the velocity of the particle in R. Explain briefly the additional terms. [Assume any formulae that you feel as necessary] [4]

b) Regard the earth as uniformly rotating reference frame with angular velocity $\vec{\omega}$. Neglecting ω^2 terms, write down the equation of motion of a projectile of mass m moving with velocity $\vec{v}$, at a point at latitude λ. Using a suitable coordinate system, obtain the general solution of this equation of motion, neglecting all ω^2-effects.

Use this result to find the location of a particle dropped at rest from a vertical tower of height H, when it reaches the ground. [1+3+2]

7. a) Calculate the moment of inertia of a rigid body about an arbitrary axis $\hat{n}$ through a given point O of the body, in terms of the inertia tensor I_{ij}. [4]

b) Find the moment of inertia of a triangular lamina about one of its sides. [3]

c) Calculate the inertia tensor for a uniform square lamina of mass M and side a about a vertex. [3]

Group – B

[*Answer* ***any two*** *questions*]

8. a) State and prove Gauss's theorem for gravitational field. [2+3]

b) What do you mean by gravitational self energy? Find an expression of gravitational self energy for a system of n particles. What will be the expression of the gravitational self-energy when all the particles will have the same mass and separated by the same average distance? [1+3+1]

9. a) Find an expression for torsional rigity of the material of a cylinder twisted through an angle θ. [5]

b) Show that for a massless cantilever the depression at any point P due to a load applied at the free end point Q is same as the depression at Q produced by a similar load at P. [5]

10. a) What is capillary ascent? Derive an expression for rise of liquid in a capillary tube of radius R. The angle of contact between the liquid and tube is θ. [1+5]

b) Two soap bubbles of radii a and b coalesce to form a single bubble of radius r. If the external pressure is P, prove that the surface tension of the soap solution is, $S = \frac{P(r^3 - a^3 - b^3)}{4(a^2 + b^2 - r^2)}$. [4]

11. a) Derive an expression for Euler's equation of motion of a fluid. [5]

b) What is Reynold's number? State its significance. [1+1]

c) A viscous liquid is flowing through a narrow horizontal tube of radius a under streamline flow. Find an expression for velocity of the liquid at a distance r from the axis of the tube. [3]

Group – C

[*Answer **any four** questions*]

12. a) Show that for small displacements x_1, x_2 and for masses m_1 and m_2 placed at positions $1/3$ distance from two ends of a uniform string of length ℓ under tension T, the eigen frequencies are given by $\omega_1 = \sqrt{\frac{3T}{m\ell}}$ and $\omega_2 = \sqrt{\frac{9T}{m\ell}}$. [4]

b) Show that the power transmitted per cycle by a progressive wave in a medium is $P = 2\pi^2\rho a^2 f^2 v$ where ρ, a, f and v are density of the medium, amplitude of vibrations, frequency of vibrations and velocity of wave respectively. [4]

c) In normal conversation, the intensity of sound is $5\times10^{-6}\,Wm^{-2}$. The amplitude and velocity of the sine wave are respectively $2\cdot4\times10^{-8}\,m$ and 332 m/s. If the density of air at STP is 1·29 kg/m. Calculate the frequency of normal human voice. [2]

13. a) Derive the equation of longitudinal waves in gases. Also discuss the Newton's formula and Laplace's formula for velocity of sound in gases. [5+2]

b) Two vibrations along the same line are described by the equations $x_1 = 0\cdot03\cos10\pi t$, $x_2 = 0\cdot03\cos12\pi t$. Find the beat frequency and beat amplitude. [3]

14. a) What do you mean by phase and group velocity of a wave? Establish the relationship between the two velocities. Calculate the group velocity of a wave whose phase velocity is proportional to the square root of the wavelength. [2+3+1]

b) Obtain an expression for apparent frequency of sound when the source and listener both are in motion assuming the wind is stationary. [4]

15. a) Find the fringe width in Young's double slit experiment where the two slits are at a distance ‘a’ and the fringe pattern is observed on a screen at a large distance ‘D’ from the plane of the slits. The wavelength of light is λ. If the irradiance of light passing through each slit be I_0, find the irradiance at the fringe maxima and minima. [5]

b) Explain what you mean by fringes of equal thickness. Explain the principle of the experiment of Newton's ring. Find the square of the radius of the p^{th} bright fringe. What is the nature of the central fringe and why? How can you measure the radius of curvature of the lens surface in contact with the flat plate used in the experiment? [5]

16. a) Find the intensity distribution as a result of Fraunhofer diffraction in a single slit of width 'b'. Sketch the pattern of intensity distribution. Find an approximate ratio of the intensity of the 1^{st}. Secondary maximum with respect to the primary maximum. [6]

b) What do you mean by the resolving power of an optical instrument? What is Rayleigh's criterion for resolution by the instrument? What is the percentage of dip in the intensity at the central part between the two intensity maxima just resolved according to Rayleigh criterion? [4]

17. a) Write down the expression of intensity pattern produced by a double slit in a Fraunhofer diffraction. If ‘b’ be the widths of the slits and ‘d’ the distance between the corresponding points of the two slits, draw the intensity pattern where d = 3b. What is missing orders and where will be the first missing order in this case? [4]

b) A point P is receiving light from a plane wavefront at a distance ‘b’ from the point. Introduce the concept of Fresnel half period zones. Find the areas of the zones for a monochromatic light of wavelength λ. Explain the variation of intensity at P as you gradually block off light from the zones starting from the first half period zone. Hence explain what you mean by the rectilinear propagation of light in the light of diffraction phenomenon. [6]

18. a) A monochromatic beam of light of wavelength λ is incident on a thin film of uniform thickness 'd' at an angle of incidence ϕ. The refractive index of the film is 'n'. Find the intensity function by the transmitted light. Explain how the sharpness of the fringe pattern formed by multiple reflection can be enhanced by manipulating the reflectance of the surfaces of the films. [5]

b) Obtain an expression for the resolving power of a plane diffraction grating. Hence find the width of a grating of 2000 lines/cm to resolve the sodium D_1 and D_2 lines in second order. [3+2]

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